

Times Series Alignment on Incomparable Spaces

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Goal 1: Heterogenous Alignments



Euclidean Points

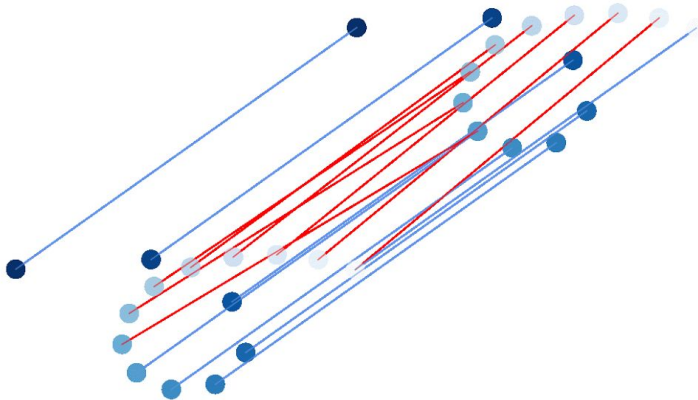


Pixel Images

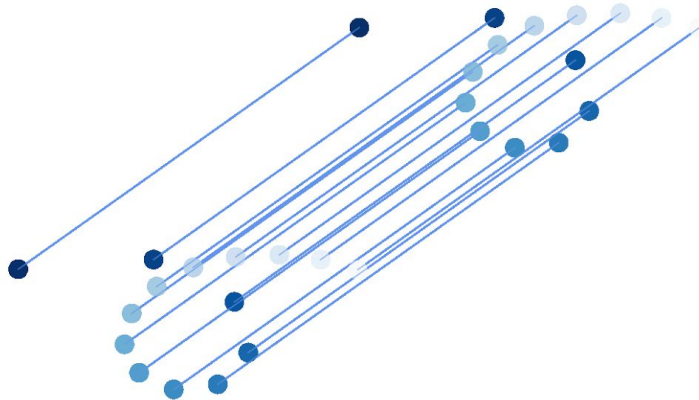
- Goal: temporal alignments between time series from different modalities

Goal 2: Invariant Alignments

DTW

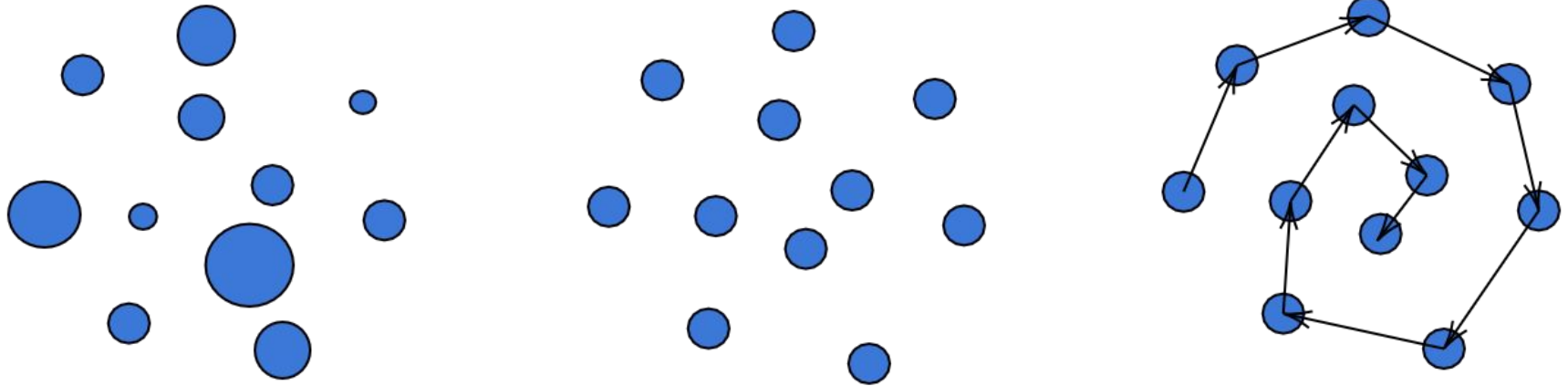


GDTW



- Goal: temporal alignments invariant to isometries (rotations, translations,...)

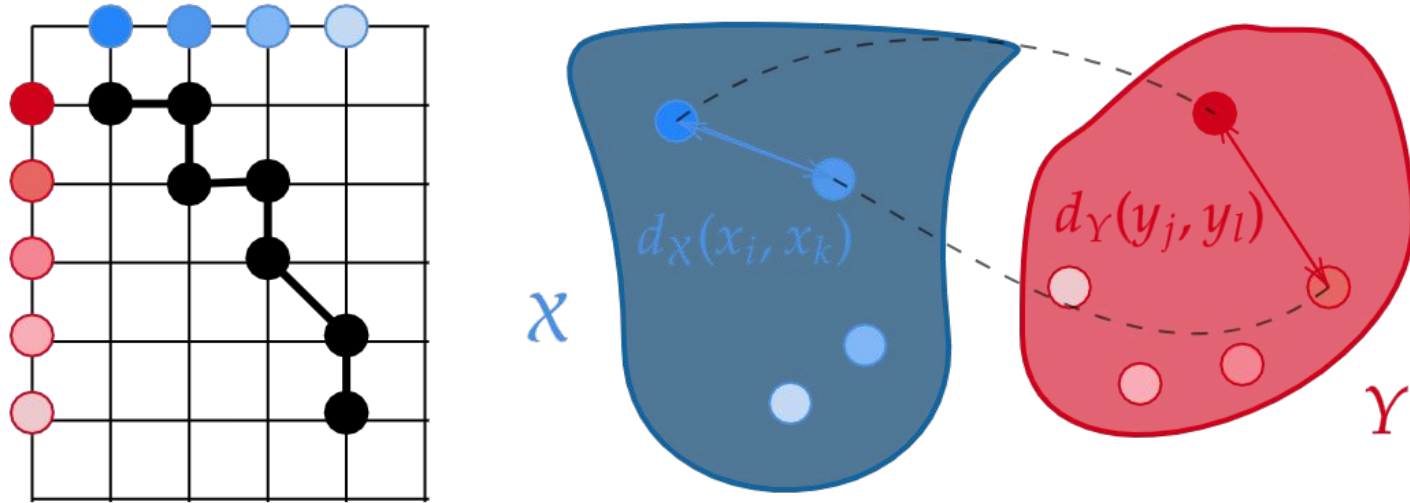
Connecting Optimal Transport with Time Series



- Time series: discrete probability measures with an ordering

Introducing Gromov Dynamic Time Warping

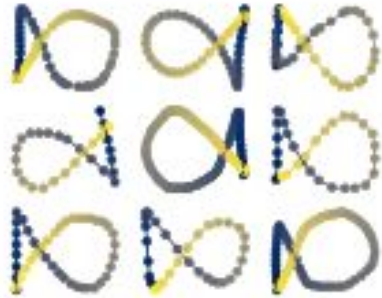
$$\text{GDTW}(\mathbf{x}, \mathbf{y}) = \min_{\mathbf{A} \in \mathcal{A}(T_x, T_y)} \sum_{ijkl} \mathcal{L} \left[d_{\mathcal{X}}(x_i, x_k), d_{\mathcal{Y}}(y_j, y_l) \right] A_{ij} A_{kl}$$



- Gromov Dynamic Time Warping: distance between time series living on potentially incomparable spaces
- Motivated by optimal transport theory

Barycentric Averaging

DTW



DTW-GI
(rotations)



DTW-GI
(rot., trans.)

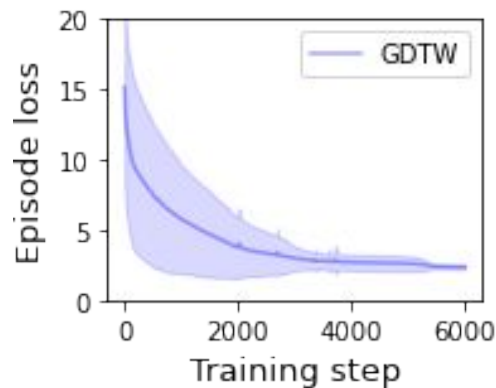
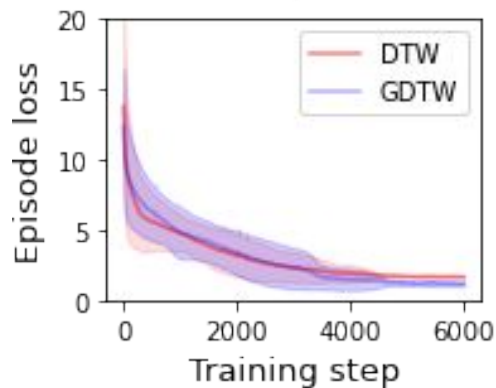


GDTW



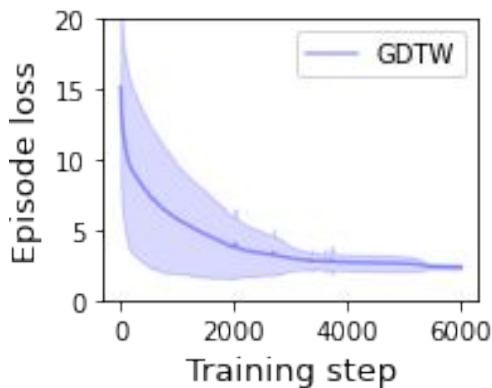
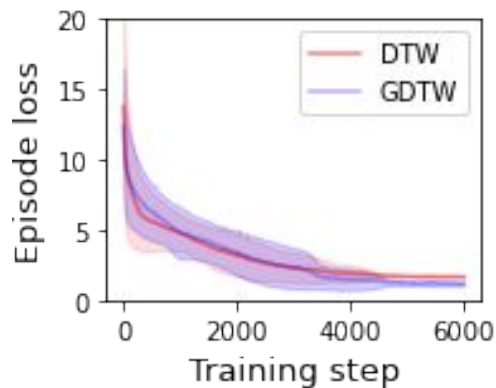
- GDTW allows averaging time series robustly thanks to invariances

Imitation Learning



- **Soft-GDTW: a soft version of GDTW allowing its use as differentiable loss**
- **Allows performing imitation learning when agent and expert are on different spaces**

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